6 Finance

Try this worksheet after you have completed section 6.6

In Chapter 6, you learned about arithmetic sequences, which have a common difference between each pair of terms, and geometric sequences, which have a common ratio between each pair of terms. You also learned that the amount of money in a bank account earning compound interest is an example of a geometric sequence. This worksheet looks at a financial situation that combines compound interest with a common difference.

When you put money in a savings account, it is unlikely that you will make no further deposits or withdrawals. The interest you earn is based on the new amount in the account each year or each month, not just on the initial investment (the principal). Similarly, when you take out a loan, you pay it back over time, and the interest is calculated on the remaining amount you owe.

EXAMPLE 1

Hayley's grandfather gives her $500 for her birthday, which she deposits in an account earning 4% interest, compounded annually. Each year on her birthday, she deposits an additional $100.

a Calculate the amount Hayley has in the account at the end of each year for the first three years.

b Write a recursive formula for the amount of money in the account.

c Write a general formula for the amount of money in the account.

d Use your general formula to find the amount of money in the account after 15 years.

Answers

a Original investment = $500

After 1 year, amount = 1.04 (500) + 100 = $620

After 2 years, amount = 1.04 (1.04 (500) + 100) + 100 = $744.80

After 3 years, amount = 1.04 (1.04 (1.04 (500) + 100) + 100) + 100 ≈ $874.59

Each year, the previous amount is multiplied by 1.04, then $100 is added.

b The recursive formula is: $u_n = u_{n-1}(1.04) + 100$, where n is the number of years.

As with all recursive formulas, you need to know the value of the previous term to find the value of the next term.

c Original investment = 500

After 1 year, amount = 1.04 (500) + 100

Look for a pattern to help you find a general formula for the value of the investment after n years.

After 2 years, amount = 1.04 (1.04 (500) + 100) +100
$= 500\,(1.04)^2 + 100\,(1.04) + 100$

Expand the brackets.

After 3 years, amount = $1.04\,(500\,(1.04)^2 + 100\,(1.04) + 100) + 100$
$= 500\,(1.04)^3 + 100(1.04)^2 + 100\,(1.04) + 100$

Factor out the 100.

After n years, amount

$= 500\,(1.04)^n + 100(1.04)^{n-1} + 100(1.04)^{n-2} + \ldots + 100\,(1.04) + 100$

$= 500\,(1.04)^n + 100\,((1.04)^{n-1} + (1.04)^{n-2} + \ldots + (1.04) + 1)$

The expression in the brackets is a geometric series with n terms, first term = 1, common ratio = 1.04

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$= 500(1.04)^n + 100\left(\frac{1.04^n - 1}{1.04 - 1}\right)$

Use the formula for geometric series $S_n = \frac{u_1(r^n - 1)}{r - 1}$ to get the general formula.

d After 15 years amount

$= 500(1.04)^{15} + 100\left(\frac{1.04^{15} - 1}{1.04 - 1}\right) \approx \$2,902.83$

Substitute $n = 15$ into the general formula.

Exercise 1

1 Ryan deposits \$200 in an account earning 2.5% interest, compounded annually. Each year, he deposits another \$100.
  a Write a general formula for the amount of money in the account after n years,
  b Use this formula to find the amount after 18 years.

2 Ari deposits \$1,000 in an account earning 1.25% interest, compounded annually. Each year, he deposits an additional \$1,000. How much money will be in the account after 10 years?

3 Maria deposits \$100 in an account earning 3% A.P.R., compounded monthly. Each month, she deposits an additional \$20.
  a Write a general formula for the amount of money in the account after n months.
  b Use this formula to find the amount of money after 5 years.

A.P.R. stands for Annual Percentage Rate.

4 Kim deposits \$50 in an account earning 1.5% A.P.R., compounded monthly. Each month, she deposits an additional \$10. How long will she have to do this to reach a balance of at least \$1,000?

5 Write a general formula for the amount of money, A, in an account after n months, if a person makes an initial deposit of P dollars, and deposits D dollars every month. The account pays interest of R%, compounded monthly.

You can use the same method to find the value remaining on a loan, where payments made decrease the amount owed.

EXAMPLE 2

The initial amount of a loan is \$5,000 and the monthly payment is \$75.
The interest rate is 6% A.P.R., compounded monthly.

a Calculate how much money is owed on the loan at the end of the first three months.
b Write a recursive formula for the amount of money owed on the loan.

Answers

a Original loan = \$5000

Amount owed after 1 month

$= 5000\left(1 + \frac{0.06}{12}\right) - 75 = 1.005(5000) - 75 = \$4,950$

Amount owed after 2 months

$4950\,(1.005) - 75 = \$4,899.75$

Amount owed after 3 months

$= 1.005\,(4899.75) - 75 \approx \$4,849.25$

b A recursive formula is $u_n = u_{n-1}(1.005) - 75$, where n is the number of months.

Each month, the previous amount owed is multiplied by 1.005, then \$75 is deducted from the amount owed.

Exercise 2

1 Write a general formula for the amount of money, A, owed on a loan after n months, if a person borrows P dollars, makes a payment of D dollars every month, and the loan carries an annual interest rate of $R\%$, compounded monthly.

2 Cheri borrows \$8,000 at an annual interest rate of 5%, compounded monthly.
She makes a payment of \$100 every month.
Find the amount remaining on the loan after 4 years.

3 Sheila takes out a mortgage loan for \$250,000 at a rate of 6% A.P.R., compounded monthly. She makes payments of \$1,500 every month.
How long will it take until the amount owed is less than \$100,000?

4 Michael borrows \$7,200 at a rate of 7.5% A.P.R., compounded monthly.
How much will he have to pay each month in order to pay back the loan in 5 years?

Chapter 6 extension worked solutions

Exercise 1

1 **a** $200\,(1.025)^n + 100\left(\frac{1.025^n - 1}{1.025 - 1}\right)$

b $200\,(1.025)^{18} + 100\left(\frac{1.025^{18} - 1}{1.025 - 1}\right) \approx \$2{,}551$

2 $1000\,(1.0125)^n + 1000\left(\frac{1.0125^n - 1}{1.0125 - 1}\right)$

$1000\,(1.0125)^{10} + 1000\left(\frac{1.0125^{10} - 1}{1.0125 - 1}\right) \approx \$11{,}714$

3 **a** monthly interest rate = (annual interest rate) ÷ 12 = $\frac{0.03}{12} = 0.0025$

$100\,(1.0025)^n + 20\left(\frac{1.0025^n - 1}{1.0025 - 1}\right)$

b 5 years = 60 months

$100\,(1.0025)^{60} + 20\left(\frac{1.0025^{60} - 1}{1.0025 - 1}\right) \approx \$1{,}409$

4 monthly interest rate = (annual interest rate) ÷ 12 = $\frac{0.015}{12} = 0.00125$

$50\,(1.00125)^n + 10\left(\frac{1.00125^n - 1}{1.00125 - 1}\right) = 1000$

Using GDC (table, graph, solver, etc), n = 90 months, or 7.5 years.

5 $$A = P\left(1 + \frac{\left(\frac{R}{100}\right)}{12}\right)^n + D\left(\frac{\left(1 + \frac{\left(\frac{R}{100}\right)}{12}\right)^n - 1}{\left(\frac{\left(\frac{R}{100}\right)}{12}\right)}\right)$$

Exercise 2

1 $$A = P\left(1 + \frac{\left(\frac{R}{100}\right)}{12}\right)^n - D\left(\frac{\left(1 + \frac{\left(\frac{R}{100}\right)}{12}\right)^n - 1}{\left(\frac{\left(\frac{R}{100}\right)}{12}\right)}\right)$$

2 monthly interest rate = (annual interest rate) ÷ 12 = $\frac{0.05}{12}$

4 years = 48 months

$$8000\left(1 + \frac{0.05}{12}\right)^{48} - 100\left(\frac{\left(1 + \frac{0.05}{12}\right)^{48} - 1}{\left(\frac{0.05}{12}\right)}\right) \approx \$4{,}465.67$$

3 monthly interest rate = (annual interest rate) ÷ 12 = $\frac{0.06}{12}$ = 0.005

$$250\,000(1.005)^n - 1500\left(\frac{(1.005)^n - 1}{0.005}\right) = 100\,000$$

Using GDC, n = 278 months, or 23 years and 2 months.

4 monthly interest rate = (annual interest rate) ÷ 12 = $\frac{0.075}{12}$ = 0.00625
5 years = 60 months

$$7200(1.00625)^{60} - D\left(\frac{((1.00625)^{60} - 1)}{0.00625}\right) = 0$$

$$D\left(\frac{((1.00625)^{60} - 1)}{0.00625}\right) = 7200(1.00625)^{60}$$

$D \approx \$144.27$