Mark scheme

Practice paper 1

1 a $r = \frac{16}{32}\left(\frac{1}{2}\right)$ (A1) (N1)

b Correct calculation or listing of terms (M1)

e.g.

$64 \times \left(\frac{1}{2}\right)^{6-1}$ *or* 64, 32, 16, 8, 4, 2

$u_6 = 2$ (A1) (N2)

c Evidence of correct substitution in S_∞ (A1)

e.g. $\frac{64}{1-\frac{1}{2}}$

$S_\infty = 128$ (A1) (N1) [5 marks]

2 a Evidence of choosing the product rule (M1)

e.g. $uv' + vu'$

Correct derivatives $-\sin x$, 2 (A1) (A1)

$f'(x) = -2x \sin x + 2 \cos x$ (A1) (N4)

b attempt to substitute into the gradient function

e.g. $f'(\pi)$ (M1)

Correct substitution

e.g. $-2(\pi)\sin(\pi) + 2\cos(\pi)$ (A1)

Gradient $= -2$ (A1) (N2) [7 marks]

3 a Evidence of a valid approach and substitution

e.g. $\frac{1}{2} \times (6)^2 \theta = 27$ (M1) (A1)

$18\theta = 27$ (A1)

$\theta = \frac{27}{18}\left(\frac{3}{2}\right)$ radians (A1) (N3)

b Evidence of correct method

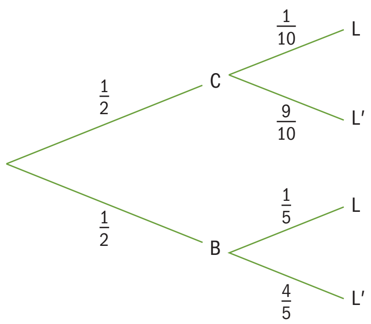
e.g. $AB = r\theta$ (M1)

$= 6 \times \frac{3}{2}$ (M1)

$= 9$ cm (A1) (N2) [7 marks]

4 **Note:** (A1) for each complementary pair.

a



(A1) (N1)

(A1) (N1)

(A1) (N1)

b Award (M1) for finding two products and for adding two products.

$\left(\frac{1}{2} \times \frac{1}{10}\right) + \left(\frac{1}{2} \times \frac{1}{5}\right)$ (M1)

$= \frac{3}{20}$ (0.15, 15%) (A1) (N1)

c Award (M1) for using the conditional probability formula.

e.g. $= \dfrac{\frac{1}{2}\times\frac{1}{5}}{\frac{3}{20}}$ (M1)

$= \dfrac{2}{3}$, (0.667) (A1) (N1) [7 marks]

5 $\cos 2\theta + 3\sin\theta = 2$ (M1)
Substitutuion of $\cos 2\theta = 1 - 2\sin^2\theta$
$1 - 2\sin^2\theta + 3\sin\theta = 2$ (M1)
Evidence of forming a quadratic
$-2\sin^2\theta + 3\sin\theta + 1 = 2$
$-2\sin^2\theta + 3\sin\theta - 1 = 0$
Multiplying through by -1 (M1)
$2\sin^2\theta - 3\sin\theta + 1 = 0$
Factoring into two binimials (A1)
$(2\sin\theta - 1)(\sin\theta - 1) = 0$
Solving for sin (A1)
$\sin\theta = \dfrac{1}{2}, 1$
Solving for θ (A1) (A2) (N2)
$\theta = \dfrac{\pi}{6}, \dfrac{\pi}{2}$ [7 marks]

6 Evidence of integration (M1)
e.g. $f(x) = \displaystyle\int \sin(4x - 1),\ \mathrm{d}x = -\frac{1}{4}\cos(4x-1)+c$ (A1) (A1)
Substituting the initial point into **their** expression (even if C is missing)
e.g. $\dfrac{3}{4} = -\dfrac{1}{4}\cos\left(4\left(\dfrac{1}{4}\right)-1\right)+c$ (M1)
$c = 1$ (A1)
$f(x) = -\dfrac{1}{4}\cos(4x-1)+1$ (A1) (N5) [6 marks]

7 a $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}\left(=\begin{pmatrix}2\\5\\3\end{pmatrix}-\begin{pmatrix}3\\2\\1\end{pmatrix}\right)$ (M1)

$\overrightarrow{AB} = \begin{pmatrix}-1\\3\\2\end{pmatrix}$ (A2) (N3)

b Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\2\\1\end{pmatrix} + t\begin{pmatrix}-1\\3\\2\end{pmatrix}$ or $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\5\\3\end{pmatrix} + t\begin{pmatrix}-1\\3\\2\end{pmatrix}$ (M1) (A1) (A1) (A1) (N4) [7 marks]

8 a Attempting to form composition (M1) (A1) (N2)
$(f \circ g)(x) = \dfrac{(x-1)+2}{x-1} = \dfrac{x+1}{x-1}$
b $x = 1$ (A1) (N1)
c Using a horizontal translation of $+3$ (M1) (A1) (N2)
$x = 4$

d Evidence of a horizontal translation of +3. (M1)
Evidence of a vertical translation of −2. (M1)
Correct values (A1)
e.g. $h(x)=\frac{(x-3)+1}{(x-3)-1}-2=\frac{x-2}{x-4}-2$ (AG) (N0)

e Recognizing the gradient of the tangent is a derivative (M1)
Evidence of using the quotient rule (M1)
Correct differentials (A1) (A1)
e.g. $h'(x)=\frac{1(x-4)-1(x-2)}{(x-4)^2}$
Finding the slope (A1) (N3)
$h'(x)=-\frac{1}{2}$ [13 marks]

9 a 51, 70 (A2) (N2)
b Substituting two points into the equation (M1) (A1)
e.g. Using (1, 1) $1 = a + b$
Using (2, 5) $5 = 4a + 2b$
Solving simultaneously (M1)
$a = 1.5$, $b = -0.5$ (A1) (A1)
$n = 1.5s^2 - 0.5s$ (AG) (N0)
c Substituting
e.g. $n = 1.5(6)^2 - 0.5(6)$ (M1) (A1) (N0)
$n = 51$
$s = 7$
$n = 1.5(7)^2 - 0.5(7)$ (A1) (N0)
$n = 70$
d Substituting and solving (M1) (A1)
$100 = 1.5s^2 - 0.5s$
$1.5s^2 - 0.5s - 100 = 0$
$s=\frac{-(-0.5)\pm\sqrt{(-0.5)^2-4\times1.5\times-100}}{2(1.5)}$
$=\frac{0.5\pm\sqrt{600.025}}{3}$
100 is not a pentagonal number as s is not an integer (R1) (N2)
e s must be an integer greater than zero (A1) (A1) (N2) [15 marks]

10 a Correct differentiation (A1) (A1)
$f'(x) = 3x^2 - 2x - 2$
Letting $x = 1$. Finding gradient (M1) (A1)
$f'(1) = 3(1)^2 - 2(1) - 2 = -1$
Finding the y value (M1) (A1)
e.g. $f(1) = (1)^3 - (1)^2 - 2(1) = -2$
Using a suitable method for finding the equation of the line (M1)
e.g. $y - (-2) = -1\,(x - (-1))$
$y = -x - 1$ (AG) (N0) [7 marks]
b Approach to find the area involving subtraction and integration (M1)
e.g. $\int f(x) - h(x),\ \int x^3 - x^2 - 2x - (-x - 1)\,dx$
Correct integral with correct signs
e.g. $\frac{x^4}{4}-\frac{x^3}{3}-\frac{x^2}{2}+x$ (A1) (A1) (A1) (A1)
Correct limits seen anywhere (A1)
e.g. $\int_{-1}^{1} x^3 - x^2 - x + 1\,dx, \left[\frac{x^4}{4}-\frac{x^3}{3}-\frac{x^2}{2}+x\right]_{-1}^{1}$
Attempt to substitute −1 and 1 (M1)
Correct substitution into **their** integral if 2 or more terms (A1)
Area = $\frac{4}{3}$ (A1) (N7) [9 marks]
[16 marks]