Practice paper 2

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1 Let $\mathbf{u} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} -12 \\ 20 \\ k \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -4 \\ 6 \\ 10 \end{pmatrix}$. *[6 marks]*

a Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are parallel. Write down the value of k.

b The angle between $\boldsymbol{u}$ and $\boldsymbol{w}$ is θ degrees. Find the value of θ.

2 Let S_n be the sum of the first n terms of an arithmetic sequence, whose first term is u_1. It is known that $S_1 = 10$, and $S_{15} = 185$. *[6 marks]*

a Write down u_1.

b Calculate the common difference of the sequence.

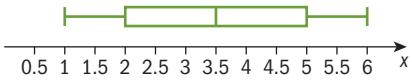
c Calculate u_{18}.

3 Consider the expansion of $(x + 3y)^{12}$. *[6 marks]*

a Write down the number of terms in the expansion.

b One term in the expansion is $ax^9 y^3$. Find the value of a.

4 A box and whisper plot and frequency distribution for a set of scores are shown below. *[7 marks]*



Score	1	2	3	4	5	6
Frequency	4	2	k	2	5	2

a Find the value of interquartile range.

b Find the value of k.

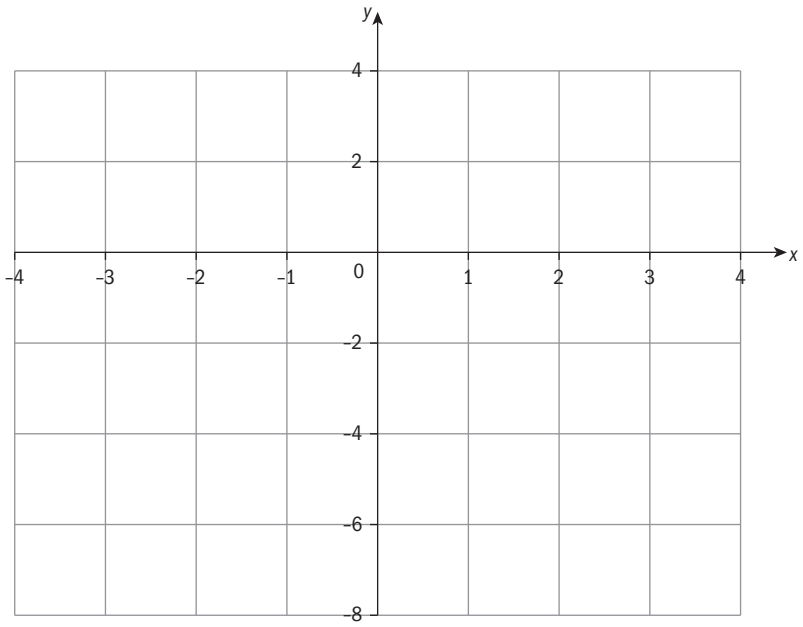
c Write down

i the standard deviation for the scores;

ii the mean for the scores.

5 Let $f(x) = x^4 - 3x^3$ for $-1 \leq x \leq 3$. *[7 marks]*

a On the grid below, sketch the graph of f.



b There is one inflexion point on the graph of f where $x > 0$.
Find the x-coordinate of this inflection point.

6 The following diagram shows triangle ABC. *[7 marks]*

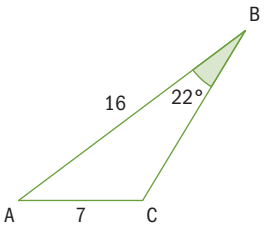


Diagram not to scale

The angle at C is obtuse, AC = 7, AB = 16 and $A\hat{B}C = 22°$.

a Find $A\hat{C}B$.

b Find the area of triangle ABC.

7 Let $f(x)=\left(\frac{1}{2}\right)^x$, $g(x) = \log_2 x$ and $h(x) = \log_2 5$. *[7 marks]*

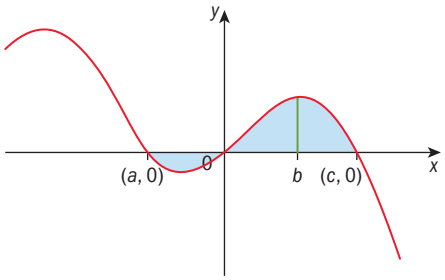
a Write down $\lim_{x\to\infty} f(x)$.

b Write down an expression for $g(x) + h(x)$ in terms of a single logarithm.

c Show that $(f \circ g)(x)=\frac{1}{x}$.

Section B

8 Let $f(x) = -x\sin(x - 2)$ for $-3 \le x \le 3$. The graph of $f(x)$ is shown below. There is a maximum point at $x = b$. The curve intersects the x-axis at the points $(a, 0)$, $(0, 0)$ and $(c, 0)$. *[15 marks]*



a Find the value of a and the value of c.

The regions between the curve and the x-axis are shaded for $a \le x \le c$ as shown.

b i Write down an expression which represents the **total** area of the shaded regions.

ii Calculate this total area.

The region between the curve and the x-axis for $0 \le x \le b$ is rotated 360° about the x-axis.

c i Find the value of b.

ii Calculate the volume of the solid.

Suppose that $f(x)$ gives the velocity of a particle moving along a straight line in ms^{-1} at x seconds.

d i Write down an expression for the distance traveled by the particle from time 0 seconds to 3 seconds.

ii Calculate the distance.

9 There are ten multiple choice questions on a science exam. Each question has 4 answer choices from which to choose. Jim felt he knew the answers to all but one question and randomly selected an answer to that question. *[13 marks]*

a Write down the probability that Jim selected a correct answer to the question for which he randomly selected an answer.

Sam did not study for the science exam and randomly chose an answer for each question.

b i Write down the expected number of correct answers on Sam's exam.

ii Find the probability that Sam randomly selected a correct answer to at least half the questions on the exam.

The scores on a history exam are normally distributed with a standard deviation of 12.3. A student must score at least a 70 to pass the exam and 80% of the students tested did not pass the exam.

c i Find the standardized value z such that $P(Z > z) = 0.80$.

ii Hence find the mean on the history exam.

If a student scores less than 90 on the exam they must take a second exam on the same material.

d Find the probability that student will **not** have to take the second exam.

10 Consider the set of data given below. *[16 marks]*

x	1	3	4	7	9	12	13	15
y	63	82	89	100	98	82	74	56

a Examine a scatterplot of the data on your GDC.

i What type of function would you choose to model the set of data? Justify your choice based on characteristics of the scatterplot.

ii Write down the equation for the type model you chose.

The amount of the moon visible to observers on Earth changes each night. During a full moon 100% of the moon is visible and during a new moon 0% is visible. The data in the table represents the percentage of the moon showing on a given day in June for the year 2009.

Day in June 2009	1	3	4	7	9	12	13	15
Percentage visible	63	82	89	100	98	82	74	56

b Comment on the appropriateness of the model you found for the data now knowing the context of the data.

Additional data for June 2009 is given in the table below.

Day in June 2009	1	3	4	7	9	12	13	15	17	19	22	25	27	30
Percentage visible	63	82	89	100	98	82	74	56	36	17	0	9	27	59

Examine a scatterplot of the data on your GDC.

c **i** What type of function would you choose to model this set of data? Justify your choice based on characteristics of the scatterplot.

ii Write down the equation for the type model you chose. Define your variables.

d The best days to stargaze are on clear nights where 10% or less of the moon is illuminated. Find the days in June 2009 where 10% or less of the moon was illuminated. Clearly show or explain how you find these days.