2 More challenging quadratics

Try this worksheet after you have completed section 2.2

In Chapter 2 you learned different techniques for solving quadratic equations.
This worksheet looks at other equations that you can solve using the same techniques.

Solving by factorizing

EXAMPLE 1

Solve the equation $x^6 - 5x^3 - 6 = 0$ by factorizing

Answer

$(x^3)^2 - 5(x^3) - 6 = 0$

Remember, $x^6 = (x^3)^2$

$(x^3 + 1)(x^3 - 6) = 0$

Now this equation is similar to a quadratic equation of the form $ax^2 + bx + c = 0$

$x^3 + 1 = 0 \rightarrow x^3 = -1 \rightarrow x = -1$

$x^3 - 6 = 0 \rightarrow x^3 = 6 \rightarrow x = \sqrt[3]{6}$

Using the zero-product property

EXAMPLE 2

Solve the equation $(2x - 3)^2 - 10(2x - 3) + 9 = 0$

Answer

$(2x - 3)^2 - 10(2x - 3) + 9 = 0$

This equation is also similar to a quadratic equation of the form $ax^2 + bx + c = 0$

$((2x - 3) - 1)((2x - 3) - 9) = 0$

$(2x - 3) - 1 = 0 \rightarrow 2x - 3 = 1 \rightarrow 2x = 4 \rightarrow x = 2$

Using the zero-product property

$2x - 3 - 9 = 0$

$2x - 12 = 0$

$2x = 12$

$x = 6$

Exercise 1

Solve these equations by factorizing.

1 $x^4 + 3x^2 - 10 = 0$

2 $2x^6 - 5x^3 + 3 = 0$

3 $x - 4\sqrt{x} + 3 = 0$

4 $3x^4 - 13x^2 + 4 = 0$

5 $(3x - 5)^2 - 2(3x - 5) - 15 = 0$

6 $(\sqrt{x} - 5)^2 + 5(\sqrt{x} - 5) - 14 = 0$

7 $\frac{2 + x^2}{x^2 + 11} = \frac{1 + 2x^2}{3x^2 - 7}$

8 $\frac{x^3 + 3}{2x^3 - 4} = \frac{x^3 + 5}{3x^3 - 9}$

Using the quadratic formula

EXAMPLE 3

Solve the equation $x^4 + 3x^2 - 5 = 0$ using the quadratic formula, giving the value of x to 3 decimal places.
Check your answer using your GDC.

Answer

$$(x^2)^2 + 3(x^2) - 5 = 0$$

$$x^2 = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$

$$x^2 = \frac{-3 + \sqrt{29}}{2}$$

$$x = \pm\sqrt{\frac{-3 + \sqrt{29}}{2}}$$

$x = 1.092$ (3 dp) or $x = -1.092$ (3 dp)

Remember, $x^4 = (x^2)^2$

Solve for x^2 using the quadratic formula, with $a = 1$, $b = 3$ and $c = -5$.

*Only use the **positive** value for x^2 here, as you can't take the square root of a negative number and get a real value.*

You can check this answer using your GDC.

Exercise 2

Solve these equations using the quadratic formula, giving the value of x to 3 decimal places.
Check your answers using your GDC.

1 $x^4 - 6x^2 - 1 = 0$

2 $2x^4 + 7x^2 - 3 = 0$

3 $-3x^6 + 2x^3 + 4 = 0$

4 $5x - 2\sqrt{x} - 4 = 0$

Chapter 2 extension worked solutions

Exercise 1

1 $x^4 + 3x^2 - 10 = 0$

$(x^2)^2 + 3(x^2) - 10 = 0$

$(x^2 + 5)(x^2 - 2) = 0$

$x^2 + 5 = 0 \rightarrow x^2 = -5 \rightarrow$ *no real root*

$x^2 - 2 = 0 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$

2 $2x^6 - 5x^3 + 3 = 0$

$2(x^3)^2 - 5(x^3) + 3 = 0$

$(x^3 - 1)(2x^3 - 3) = 0$

$x^3 - 1 = 0 \rightarrow x^3 = 1 \rightarrow x = 1$

$2x^3 - 3 = 0 \rightarrow x^3 = \frac{3}{2} \rightarrow x = \sqrt[3]{\frac{3}{2}}$

3 $x - 4\sqrt{x} + 3 = 0$

$(\sqrt{x})^2 - 4(\sqrt{x}) + 3 = 0$

$(\sqrt{x} - 1)(\sqrt{x} - 3) = 0$

$\sqrt{x} - 1 = 0 \rightarrow \sqrt{x} = 1 \rightarrow x = 1$

$\sqrt{x} - 3 = 0 \rightarrow \sqrt{x} = 3 \rightarrow x = 9$

4 $3x^4 - 13x^2 + 4 = 0$

$3(x^2)^2 - 13(x^2) + 4 = 0$

$(x^2 - 4)(3x^2 - 1) = 0$

$x^2 - 4 = 0 \rightarrow x^2 = 4 \rightarrow x = \pm 2$

$3x^2 - 1 = 0 \rightarrow x^2 = \frac{1}{3} \rightarrow x = \pm\sqrt{\frac{1}{3}}$

5 $(3x - 5)^2 - 2(3x - 5) - 15 = 0$

$((3x - 5) - 5)((3x - 5) + 3) = 0$

$(3x - 5) - 5 = 0 \rightarrow 3x - 5 = 5 \rightarrow 3x = 10 \rightarrow x = \frac{10}{3}$

$(3x - 5) + 3 = 0 \rightarrow 3x - 5 = -3 \rightarrow 3x = 2 \rightarrow x = \frac{2}{3}$

6 $(\sqrt{x} - 5)^2 + 5(\sqrt{x} - 5) - 14 = 0$

$((\sqrt{x} - 5) + 7)((\sqrt{x} - 5) - 2) = 0$

$\sqrt{x} - 5) + 7 = 0 \rightarrow \sqrt{x} - 5 = -7 \rightarrow \sqrt{x} = -2 \rightarrow$ *no real root*

$(\sqrt{x} - 5) - 2 = 0 \rightarrow \sqrt{x} - 5 = 2 \rightarrow \sqrt{x} = 7 \rightarrow x = 49$

7 $\frac{2 + x^2}{x^2 + 11} = \frac{1 + 2x^2}{3x^2 - 7}$

$(2 + x^2)(3x^2 - 7) = (x^2 + 11)(1 + 2x^2)$

$3x^4 - x^2 - 14 = 2x^4 + 23x^2 + 11$

$x^4 - 24x^2 - 25 = 0$

$(x^2)^2 - 24(x^2) - 25 = 0$

$(x^2+1)(x^2-25)=0$

$x^2+1=0 \rightarrow x^2=-1 \rightarrow$ *no real root*

$x^2-25=0 \rightarrow x^2=25 \rightarrow x=\pm 5$

8 $\dfrac{x^3+3}{2x^3-4}=\dfrac{x^3+5}{3x^3-9}$

$(x^3+3)(3x^3-9)=(2x^3-4)(x^3+5)$

$3x^6-27=2x^6+6x^3-20$

$x^6-6x^3-7=0$

$(x^3)^2-6(x^3)-7=0$

$(x^3-7)(x^3+1)=0$

$x^3-7=0 \rightarrow x^3=7 \rightarrow x=\sqrt[3]{7}$

$x^3+1=0 \rightarrow x^3=-1 \rightarrow x=\sqrt[3]{-1}=-1$

Exercise 2

1 $(x^2)^2-6(x^2)-1=0$

$x^2=\dfrac{6\pm\sqrt{(-6)^2-4(1)(-1)}}{2(1)}$

$x^2=\dfrac{6+\sqrt{40}}{2}$

$x=\pm\sqrt{\dfrac{6+\sqrt{40}}{2}}=\pm 2.482$ (3 dp)

2 $2(x^2)^2+7(x^2)-3=0$

$x^2=\dfrac{-7\pm\sqrt{(7)^2-4(2)(-3)}}{2(2)}$

$x^2=\dfrac{-7+\sqrt{73}}{4}$

$x=\pm\sqrt{\dfrac{-7+\sqrt{73}}{4}}=\pm 0.621$ (3 dp)

3 $-3(x^3)^2+2(x^3)+4=0$

$x^3=\dfrac{-2\pm\sqrt{(2)^2-4(-3)(4)}}{2(-3)}$

$x^3=\dfrac{-2\pm\sqrt{52}}{-6}=\dfrac{2\pm\sqrt{52}}{6}$

$x=\sqrt[3]{\dfrac{2+\sqrt{52}}{6}}=1.154$ (3 dp) and $x=\sqrt[3]{\dfrac{2-\sqrt{52}}{6}}=-0.954$ (3 dp)

4 $5(\sqrt{x})^2-2(\sqrt{x})-4=0$

$\sqrt{x}=\dfrac{2\pm\sqrt{(-2)^2-4(5)(-4)}}{2(5)}$

$\sqrt{x}=\dfrac{2\pm\sqrt{84}}{10}$

$x=\left(\dfrac{2+\sqrt{84}}{10}\right)^2=1.247$ (3 dp)

$x=\left(\dfrac{2-\sqrt{84}}{10}\right)^2=0.513$ (3 dp)