Mark scheme

Practice paper 2

1 a $k = -32$ (A1) (N1)

b Calculating $u \cdot w, |u|, |w|$ (A1) (A1) (A1)

e.g. $u \cdot w = (3)(-4) + (-5)(6) + 8(10)$ $(= 38)$

$|u| = \sqrt{(3)^2 + (-5)^2 + (8)^2}\ \left(= \sqrt{98}\right)$

$|w| = \sqrt{(-4)^2 + (6)^2 + (10)^2}\ \left(= \sqrt{152}\right)$

Evidence of using the formula to find the angle (M1)

e.g. $\cos\theta = \dfrac{(3)(-4)+(-5)(6)+8(10)}{\sqrt{(-4)^2+(6)^2+(10)^2}\sqrt{(-4)^2+(6)^2+(10)^2}}, \dfrac{38}{\sqrt{98}\sqrt{152}}$

$\theta \approx 71.9°$ (A1) (N3) [6 marks]

2 a $u_1 = S_1$ (R1)

$u_1 = 10$ (A1) (N2)

a Evidence of substituting into $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ (M1)

e.g. $185 = \frac{15}{2}[2(10) + (15-1)d]$

$d = 0.333 \left(= \frac{1}{3}\right)$ (A1) (N1)

b Evidence of substituting into $u_n = u_1 + (n - 1)d$ (M1)

e.g. $u_{18} = 10 + (18 - 1)(0.333)$

$u_{18} = 15.7 \left(= \frac{47}{3}\right)$ (A1) (N1) [6 marks]

3 a 13 (A1) (N1)

b Identifying the required term (seen anywhere) (M1)

e.g. $\binom{12}{9}(x)^9(3y)^3$

$\binom{12}{9} = 220$ (A1)

$27y^3$, 3^3, 27 (A1)

$a = 5940$ (A2) (N3) [6 marks]

4 a $Q_1 = 2$ and $Q_2 = 5$ (A1) (A1)

IQR = 3 (A1) (N3)

b Recognizing that the number of data points greater than or equal to 4 must be equal to the number of data points less than or equal to 3 (M1)

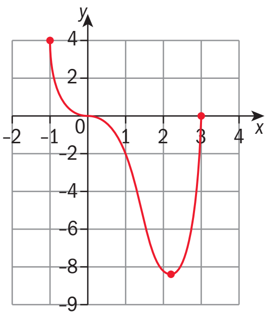
e.g. $4 + 2 + k = 2 + 5 + 2$, median = 3.5

$k = 3$ (A1) (N2)

c i standard deviation = 1.74 (A1)

ii mean = 3.44 (A1) (N2) [7 marks]

5 a (A1) (A1) (A1) (N3)



Note: Award **A1** for approximately correct shape, **A1** for correct endpoints and **A1** for relative minimum point.

b $f'(x) = 4x^3 - 9x^2$, $f''(x) = 12x^2 - 18x$ (A1) (A1)

Evidence of setting second derivative equal to 0 (M1)

e.g. $12x^2 - 18x = 0$, $f''(x) = 0$

$x = 1.5$ (A1) (N0) [7 marks]

6 a Evidence of using sine rule (M1)
Correct substitution into sine rule (A1)
e.g. $\frac{\sin C}{16} = \frac{\sin 22°}{7}$
Attempt to solve (M1)
e.g. $\sin C = \frac{16\sin 22°}{7}$, $\sin C = 0.8562436...$
C = 58.897 (A1)
$A\hat{C}B = 180 - 58.897 \ldots = 121.1026\ldots = 121°$ (A1) (N3)
b correct substitution into the formula for the area of a triangle (A1)
e.g. $\text{Area} = \frac{1}{2}(6)(16)\sin(121.1026...)$
Area = 41.1 (A1) (N2) [7 marks]

7 a 0 (A1) (N1)
b Write an expression for $g(x) + h(x)$ (M1)
e.g. $\log_2 x + \log_2 5$
Correctly applies the property $\log_c a + \log_c b = \log_c ab$ (A1) (N2)
$\log_2 (5x)$
c Attempt to form composition (M1)
e.g. $(f \circ g)(x) = f(\log_2 x)$
Correct answer (A1) (N2)
$\left(\frac{1}{2}\right)^{\log_2 x}$
Find a common base (M1)
e.g. $\left(\frac{1}{2}\right)^{\log_2 x} = \left(2^{-1}\right)^{\log_2 x} = 2^{-\log_2 x}$
Correctly apply the property $\log_c a^r = r\log_c a$ (M1)
e.g. $2^{\log_2 x^{-1}} = 2^{\log_2 \frac{1}{x}}$
Correctly apply the property $\log_a a^x = x = a^{\log_a x}$ (A1) (N0) [7 marks]
$2^{\log_2 \frac{1}{x}} = \frac{1}{x}$

8 a Evidence of valid approach (M1)
e.g. $f(x) = 0$
$a = -1.14$ (or $2-\pi$), $c = 2$ (A1) (A1) (N3) [3 marks]
b Valid approach recognizing two regions (M1)
e.g. finding two areas
Correct working (A1)
e.g. $-\int_{-1.14159...}^{0}(-x\sin(x-2))\,dx + \int_{0}^{2}(-x\sin(x-2))\,dx$
Area = 1.32 (A2) (N3) [4 marks]
c i Attempt to find b (M1)
e.g. setting $f'(x) = 0$, graph
$b = 1.15$ (A1) (N2)
ii Attempt to substitute either limits or function into formula (M1)
e.g. $V = \pi\int_{0}^{1.15}\left(f(x)\right)^2 dx$
Volume = 1.26 (accept 1.25) (A2) (N2) [5 marks]
d i Attempt to substitute either limits or function into formula (M1)
e.g. $\text{distance} = \int_{0}^{3}|f(x)|\,dx$
ii Distance = 2.31 m (A2) (N2) [3 marks]
Total [15 marks]

9 a $\frac{1}{4}$ or 0.25 (A1) (N1) [1 mark]

b i $10\left(\frac{1}{4}\right)=2.5$ (A1)

ii Evidence of appropriate approach involving binomial (M1)
e.g. $X \sim B(10, 0.25)$
Realizing Sam has to answer 5 or more questions correctly (A1)
e.g. $P(X \geq 5)$
Valid approach (M1)
e.g. $1 - P(X \leq 4)$
P(at least half correct) = 0.0781 (A1) (N3) [5 marks]

c i Evidence of approach (M1)
e.g. $P(Z \leq z) = 0.20$
$x = -0.841621.....$ (A1) (N2)
$= -0.842$

ii Evidence of appropriate approach (M1)
e.g. $\frac{x-\mu}{\sigma} = -0.841621...$
Correct substitution (A1)
e.g. $\frac{70-\mu}{12.3} = -0.841621...$
$\mu = 80.4$ (A1) (N1) [5 marks]

d Evidence of approach (M1)
e.g. $P(X \geq 90)$
$P(X \geq 90) = 0.218$ (accept 0.216) (A1) (N2) [2 marks]
Total [13 marks]

10 a i Choosing a quadratic model (A1)
Giving appropriate justification (R1) (N0)
e.g. there appears to be one vertex, the function increases and then decreases

ii $y = -0.819x^2 + 12.4x + 52.0$ (A3) (N3) [5 marks]
Note: one mark for each parameter

b Giving appropriate reasoning
e.g. The model is not appropriate because the visibility of the moon will not decrease without bound. It will always be from 0% to 100%. (R1) (N1) [1 mark]

c i Choosing correct type function (A1)
e.g. sine, cosine
Giving appropriate justification (R1) (N2)
e.g. the data is periodic

ii Correct model function (A3)
e.g. $p(x) = 49.3 \sin(0.213x - 0.479) + 53.5$
Note: A3 for 4 correct parameters, A2 for 3 correct parameters, A1 for 2 correct parameters
Define variables (A2) [7 marks]
e.g. x = day in June 2009 and p = percentage of moon visible

d Evidence of correct approach
e.g. $49.3 \sin(0.213x - 0.0479) + 53.5 = 0$, (M1)
$49.3 \sin(0.213x - 0.0479) + 53.5 \leq 10$
Solves equation or inequality (A1)
e.g. $x = 20.1, 24.7, 20.1 \leq x \leq 24.7$
(Accept $x = 20.0, 24.6$)
The best days to star gaze are June 20th through to June 24th. (A1) (N0) [3 marks]
(Accept June 20th through to June 25th)
Total [16 marks]