5 Continued fractions and more asymptotes

Try this worksheet after you have completed Exercise 5D.

Investigating continued fractions

Here are some terms in a sequence of continued fractions.

Term 1 = 3

Term 2 = $3+\frac{1^2}{6}$

Term 3 = $3+\cfrac{1^2}{6+\cfrac{3^2}{6}}$

Term 4 = $3+\cfrac{1^2}{6+\cfrac{3^2}{6+\cfrac{5^2}{6}}}$

Term 5 = $3+\cfrac{1^2}{6+\cfrac{3^2}{6+\cfrac{5^2}{6+\cfrac{7^2}{6}}}}$

Exercise 1

1 Copy the table and evaluate terms 1 to 5. Find expressions for terms 6 and 7, then evaluate them to complete the table. Use a GDC for the calculations and show the full display from your screen in the value column.

Term number (x)	Value (y)
1	3
2	
3	3.133333333
4	
5	
6	
7	

2 Predict the number to which the sequence converges.

3 Sketch a graph of the function with $1 \le x \le 7$ and $3 \le y \le 3.2$

Oblique asymptotes

You need to complete Extension worksheet 1, section A, Dividing polynomials, before you try this activity.

In Chapter 5 you studied horizontal and vertical asymptotes.

Oblique asymptotes are often called ‘slant asymptotes’. An oblique asymptote is a linear asymptote that is not parallel to the x- or y-axis.

For a graph with equation $\frac{f(x)}{g(x)}$, if the degree of the numerator, $f(x)$, is bigger than the degree of the denominator, $g(x)$, then the graph has an oblique asymptote.

The degree of an expression is its highest power.

To find an oblique asymptote, divide the numerator by the denominator.

The quotient is the oblique asymptote; ignore any remainder.

EXAMPLE 1

Investigate the function $f(x)=\dfrac{2x^2+1}{x-1}$

a Use your GDC to help you sketch $f(x)$.

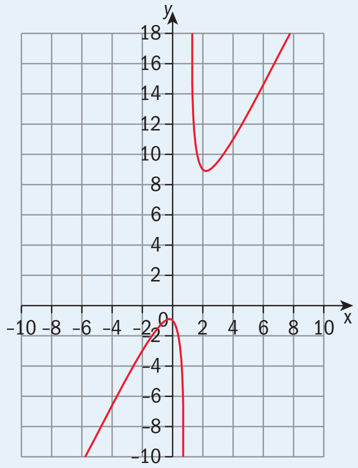
b Use long division to find the oblique asymptote.

c Draw the oblique asymptote on your sketch.

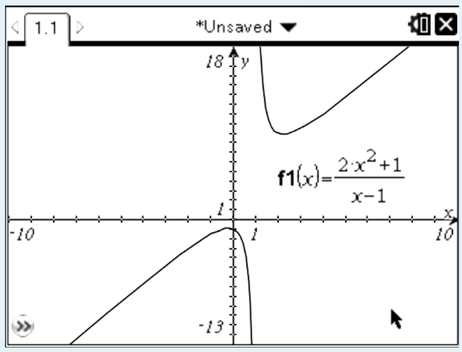
> The numerator has degree 2 and the denominator, degree 1. This function has an oblique asymptote.

Answers

a



Use the GDC to draw the graph of f(x)



b

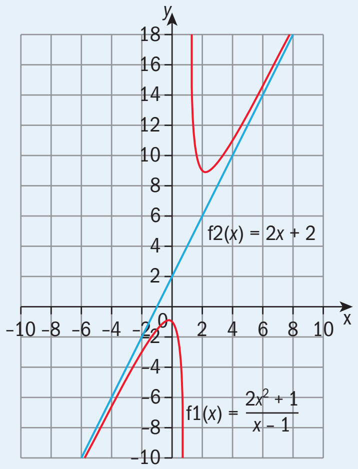
$$x-1\,\begin{array}{|l}\quad 2x\ \ +2\\ \hline 2x^2+0x+1\\ \underline{2x^2-2x}\\ \qquad\quad 2x+1\\ \qquad\quad \underline{2x-2}\\ \qquad\qquad\ \ 3\end{array}$$

The oblique asymptote is $y = 2x + 2$

$y = 2x + 2$ is the equation of the oblique asymptote.

Ignore remainder. It shows that, whatever x-value you use, you will never reach $2x + 2$ for f(x).

c



Draw the line $y = 2x + 2$ on your sketch.

Exercise 2

For each function

i find the oblique asymptote

ii sketch the graph and draw the oblique asymptote on it.

a $f(x)=\dfrac{x^2+6x+8}{x+1}$ **b** $f(x)=\dfrac{x^2-3x-4}{x-2}$ **c** $f(x)=\dfrac{x^2-4x+1}{x+3}$

d $f(x)=\dfrac{x^2+1}{x-1}$ **e** $f(x)=\dfrac{x^2+2x-2}{x+2}$ **f** $f(x)=\dfrac{2-3x^2}{x-1}$

Chapter 5 extension worked solutions

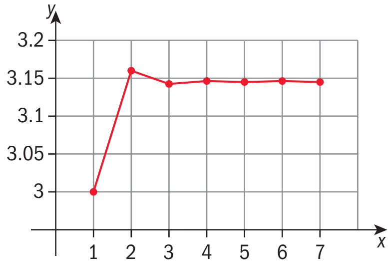
Exercise 1

1

Term number (x)	Value (y)
1	3
2	3.166666667
3	3.133333333
4	3.145238095
5	3.13968254
6	3.142712843
7	3.140881341

2 The sequence is converging to π

3



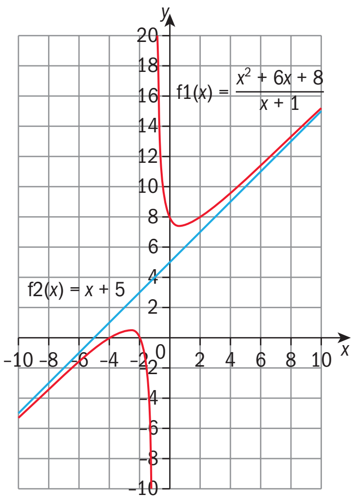
Exercise 2

a **i**

$$\begin{array}{r|l} & \;x\;+5 \\ \cline{2-2} x+1 & x^2+6x+8 \\ & \underline{x^2+\;x} \\ & \quad\;\;5x+8 \\ & \quad\;\;\underline{5x+5} \\ & \qquad\quad 3 \end{array}$$

The oblique asymptote is $y = x+5$

ii

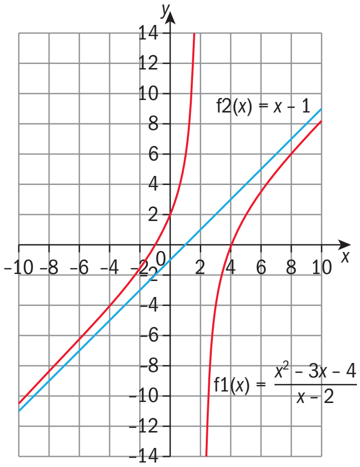


b i

$$\begin{array}{r|l} & \quad x \;\; -1 \\ \cline{2-2} x-2 & x^2-3x-4 \\ & x^2-2x \\ \cline{2-2} & \quad -\;\; x-4 \\ & \quad -\;\; x+2 \\ \cline{2-2} & \qquad\quad -6 \end{array}$$

The oblique asymptote is $y = x-1$

ii

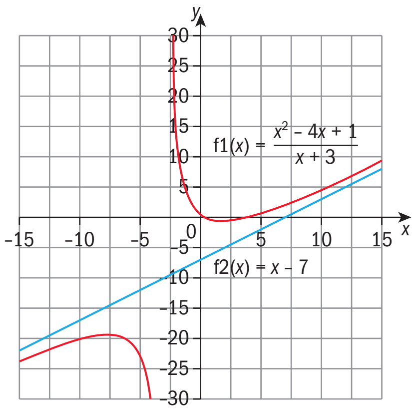


c i

$$\begin{array}{r|l} & \quad x \;\; -7 \\ \cline{2-2} x+3 & x^2-4x+1 \\ & x^2+3x \\ \cline{2-2} & \quad -7x+\;\;1 \\ & \quad -7x-21 \\ \cline{2-2} & \qquad\quad 22 \end{array}$$

The oblique asymptote is $y = x-7$

ii

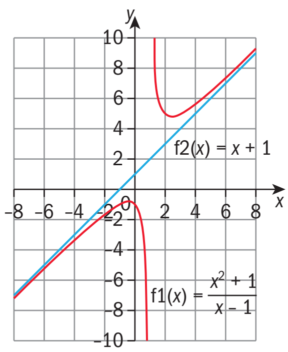


d i

$$\begin{array}{r|l} & \quad x \;\; +1 \\ \cline{2-2} x-1 & x^2+0x+1 \\ & x^2-\;x \\ \cline{2-2} & \qquad\quad x+1 \\ & \qquad\quad x-1 \\ \cline{2-2} & \qquad\qquad 2 \end{array}$$

The oblique asymptote is $y = x+1$

ii

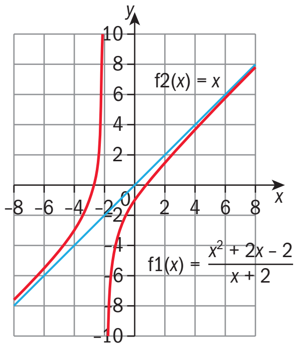


e i

$$\begin{array}{r|l} & \quad x \\ \cline{2-2} x+2 & x^2+2x-2 \\ & x^2+2x \\ \cline{2-2} & \qquad\quad -2 \end{array}$$

The oblique asymptote is $y = x$

ii



f i

$$\begin{array}{r|l} & -3x \quad -3 \\ \cline{2-2} x-1 & -3x^2+0x+2 \\ & -3x^2+3x \\ \cline{2-2} & \qquad -3x \;\; +2 \\ & \qquad -3x \;\; +3 \\ \cline{2-2} & \qquad\qquad -1 \end{array}$$

The oblique asymptote is $y = -3x - 3$

ii